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SUPERBANANA FOKKER-PLANCK EQUATION FOR TOKAMAKS WITH THE STRONG TOROIDAL FIELD RIPPLES

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The Fokker-Planck description of collisional ripple transport processes of fast ions is extended for tokamaks with strong toroidal field (TF) ripples. The topology of superbanana orbits generated by the TF ripple drift of banana "guiding" centers is analyzed in terms of the adiabatic invariant. The transport coefficients of a 4D Fokker-Planck equation are derived for the case of strong TF ripples. This study aims at a generalization of the kinetic simulation of fast ions in plasmas of present-day and next generation tokamaks.
KEY WORDS: 3D Fokker-Planck equation, toroidal field ripples, COM space, superbanana orbits, alpha particle losses.

СУПЕРБАНАНОВОЕ УРАВНЕНИЕ ФОККЕРА-ПЛАНКА В СЛУЧАЕ СИЛЬНОЙ ГОФРИРОВКИ МАГНИТНОГО ПОЛЯ

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Описание столкновительного гофрировочного переноса быстрых ионов при помощи уравнения Фоккера-Планка, распространено на область сильной гофрировки тороидального поля (ТП) токамака. Проанализирована топология супербанановых орбит, образованных вследствие дрейфа ведущих центров бананов в гофрах ТП, с использованием адиабатического инварианта. Получены транспортные коэффициенты уравнения Фоккера-Планка в четырёх мерном фазовом пространстве при наличии сильной гофрировки ТП. Исследование выполнено в рамках обобщения метода кинетического моделирования быстрых ионов, в плазме современных и будущих токамаков.
КЛЮЧЕВЫЕ СЛОВА: 3-х мерное уравнение Фоккера-Планка, гофры тороидального магнитного поля, пространство инвариантов движения, супербанановые орбиты, потери альфа частиц.

СУПЕРБАНАНОВЕ РІВНЯННЯ ФОККЕРА-ПЛАНКА ПРИ НАЯВНОСТІ СИЛЬНОГО ГОФРУВАННЯ МАГНІТНОГО ПОЛЯ

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Описання гофрованого переносу швидких іонів, обумовленого зіткненнями, за допомогою рівняння Фоккера-Планка, поширене на область сильного гофрування тороїдального поля (ТП) токамака. Проаналізована топологія супербананових орбіт частинок, які утворені внаслідок дрейфу ведучих центрів бананів у гофрах ТП, з використанням адіабатичного інваріанту. Отримані транспортні коефіцієнти рівняння Фоккера-Планка у чотирьох вимірному фазовому просторі за наявності сильного гофрування ТП. Дослідження проведено в рамках узагальнення методу кінетичного моделювання швидких іонів в плазмі сучасних та майбутніх токамаків.
КЛЮЧОВІ СЛОВА: 3-вимірне рівняння Фоккера-Планка, гофри тороїдального магнітного поля, простір інваріантів руху, супербананові орбіти, втрати альфа частинок.

The confinement of energetic ions such as fusion produced α-particles is essential to sustain burning plasma conditions. Tokamak experiments on charged fusion product confinement and direct loss measurements [2, 3] gave rise to intensive theoretical investigations on alpha particle behavior in tokamak plasmas [1, 4-10].

While in many studies the tokamak has been associated with an axisymmetric configuration, the real toroidal magnetic field lines will exhibit undulations, referred to as ripples. There will be two main sources for forming TF ripples in ITER. First, the toroidal magnetic field is created by the finite number (N=18) of toroidal field coils with spaces between them large enough to accommodate ports. The magnetic field ripples generated by the discrete coil assembly will have toroidal mode numbers nN with n=1,2,… , where n=1 is the dominant mode. Placing ferritic inserts underneath the toroidal field coils can mitigate the amplitude of these ripples to a reasonable magnitude of magnetic perturbations. Second, when the test blanket modules (TBM) will be installed, their ferromagnetic materials are expected to significantly perturb the local toroidal field.

TF ripples can enhance particle losses, most consequential for energetic ion confinement [1, 9, 11-17], commonly referred to as 'ripple losses'. Obviously, the resulting heat load on the first wall limits the allowable TF ripple amplitude in a tokamak fusion reactor, thereby setting a lower limit on the number of toroidal coils. Usually, the TF ripple amplitude is defined as $\delta \equiv (B_{t\max} - B_{t\min})/(B_{t\max} + B_{t\min})$, where $B_{t\max}$ and $B_{t\min}$ are the magnitudes of toroidal magnetic fields calculated at two points having the same radial and vertical coordinates but different toroidal position – one ($B_{t\max}$) in the center of a TF coil and the other ($B_{t\min}$) right in the middle between two neighboring coils.

Ripple losses are numerically predictable as confirmed by experiments e.g. in JET [18, 19], TFTR [3, 6, 8] and Tore Supra [12]. As enhanced alpha particle losses can constitute a critical problem in future tokamak reactors, significant effort has been dedicated to reducing the TF ripple magnitude also in ITER [20-22]. The development of operational scenarios for next-generation tokamaks demands for reliable description of TF ripple induced transport of charged fusion products as well as of ions generated by neutral beam injection.

The effect of the ripple collisional transport on the fast ions in tokamaks can be adequately described by the Fokker-Planck equation in 3D constant of motion (COM) space [1, 4-7, 23]. The transport coefficients of this equation in the case of weak ripples, $\delta << \delta_1 = \varepsilon/(Nq)^{3/2}$ where ε is the flux surface toroidicity and q the safety factor, were derived in [1]. For moderate-energy ions at the plasma periphery and for high-energy ions in the plasma core, where usually δ does not exceed the Goldston-White-Boozer stohasticity threshold δ_{GWB} [12], TF ripples may result in a substantial enlargement of collisional radial transport of toroidally trapped particles if they resonate with the ripple perturbations. 3D Fokker-Planck confinement modeling of MeV charged fusion products has shown that collisional ripple transport of resonant bananas, which are called *superbananas* and satisfy $l\omega_b - N\omega_d = 0,\ \ l = 0, \pm 1, \pm 2, ...\infty$ where ω_b is the particle bounce frequency and ω_d the toroidal precession frequency, may be responsible for the delayed loss of partially thermalized charged fusion products observed in TFTR [6, 8].

In the present paper we are primarily interested in the kinetic description of classical transport processes (induced by collisions and ripple orbital effects) of fast ions in tokamak plasmas. The consideration of ripples makes the bounce-averaged kinetic equation 4-dimensional. Here we use a 4D phase space spanned by 3 constants of motion plus an angular coordinate determining the position on a superbanana orbit. First of all, the transport coefficients of this 4D Fokker-Planck equation are quantitatively derived for the general case of arbitrary ripple magnitudes, i.e. including strong TF ripples.

The outline of this paper is as follows. In section SUPERBANANA ORBITS, we investigate superbanana orbits. The COM space variables suitable for superbanana averaging of the kinetic equation are defined in section COM SPACE VARIABLES. The explicit form of the superbanana Fokker-Planck kinetic equation in the case of strong TF ripples is obtained in section SUPERBANANA FOKKER-PLANCK EQUATION. Finally, conclusions are drawn in section SUMMARY AND DISCUSSION.

SUPERBANANA ORBITS

This section is devoted to the investigation of the banana averaged motion in the vicinity of the *l*-th resonance. This motion can be treated as the behavior of 1D system with canonical variables (p, ψ) and Hamiltonian h in the form

$$h = p^2/2 - M\cos(\xi + p)\cos\psi\,, \tag{1}$$

where p is the generalized momentum representing the normalized toroidal momentum, ψ is conjugate coordinate characterizing the position in toroidal angle. M and ξ are considered as constant parameters of 1D system. The explicit expression for p, ψ, M, ξ and their relationship with the parameters of particle and magnetic configuration are defined in [1].

Assuming that $M >> 1$ equation (1) for Hamiltonian h can be present in the form

$$h = -M\cos(\xi + p)\cos\psi\,. \tag{2}$$

Let us start investigation of evolution 1D system with Hamiltonian h in the form (2) from defining the stagnation points. The coordinates of these points are given by equations $\partial_p h = 0, \partial_\psi h = 0$ i.e.

$$M\sin(\xi + p)\cos\psi = 0\,, \tag{3}$$

$$M\cos(\xi + p)\sin\psi = 0\,. \tag{4}$$

These equations have two groups of roots *i*-th and *j*-th

$$\begin{cases} \psi_{i_1} = \pi/2 + \pi i_1,\ i_1 \in \mathbb{Z} \\ p_{i_2} = \pi/2 - \xi + \pi i_2,\ i_2 \in \mathbb{Z} \end{cases}, \tag{5}$$

$$\begin{cases} \psi_{j_1} = \pi j_1, \; j_1 \in \mathbb{Z} \\ p_{j_2} = -\xi + \pi j_2, \; j_2 \in \mathbb{Z} \end{cases}. \tag{6}$$

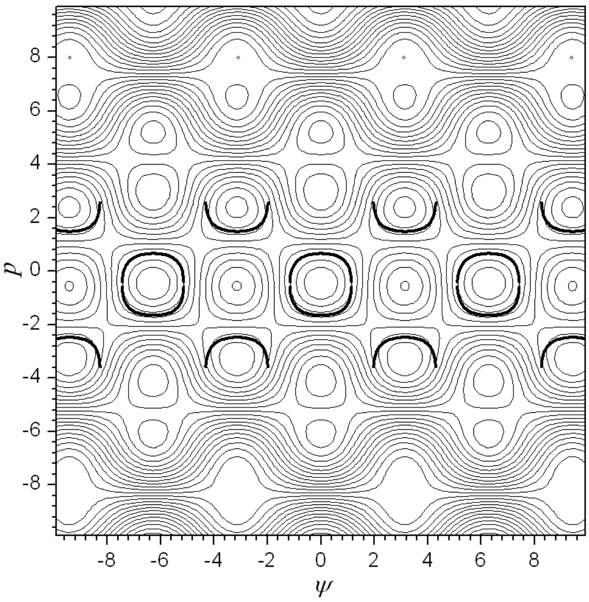

Fig. 1. Contour plot of Hamiltonian $h(M=10)$ given by Eq.(1) and approximate orbit for $M>>1$
$p = -\xi \pm \arccos(-h/M\cos(\psi)) + 2\pi k, k \in \mathbb{Z}$

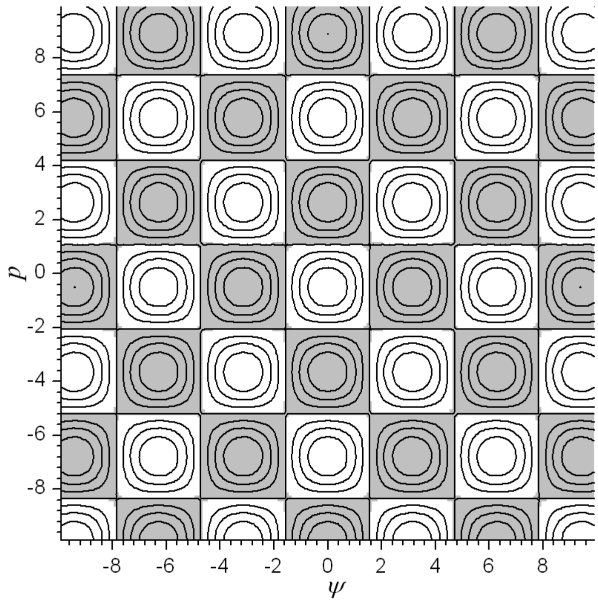

Fig. 2. Contour plot of Hamiltonian h given by Eq.(2) for $M>>1$.

Analyzing the determinant

$$D = \frac{\partial^2 h}{\partial \psi^2}\frac{\partial^2 h}{\partial p^2} - \left(\frac{\partial^2 h}{\partial p \partial \psi}\right)^2, \tag{7}$$

it is seen that *i*-th group of roots are X-points, because $D_i < 0$, and *j*-th group of roots are O-points, because $D_i > 0$. It should be pointed out that O-points with $j_1 + j_2 = 2k$, where $k = 0, \pm 1, \ldots$, correspond to minimum h (= $-M$) and O-points with $j_1 + j_2 = 2k+1$ correspond respectively to maximum h (= M).

It is seen from Fig.1 and Fig.2 that at $M \geq 10$ the reduced Hamiltonian h given by Eq.(2) h is in a good accordance with the rigorous one (Eq.(1)) in the vicinity of the resonant level $|p| < M$. At $|p| \ll 1$ there are two types of domains with the closed orbits. These are domains with $h<0$ and domains with $h>0$. It should be noted, that at $M>>1$ the reduced Hamiltonian equals zero on separatrix. Thus, analysis of the superbanana motion in the resonant region is restricted to considering only orbits in these two domains.

COM SPACE VARIABLES

Following the procedure proposed in [1] one can introduce pair of new variables $\left(p_{\beta\max}, \vartheta_3\right)$ connected with (p, ψ). According to [1] $p_{\beta\max}$ can be treated as the maximum radial coordinate along the superbanana orbit and ϑ_3 could be considered as the cyclic variable. The subscript '3' indicates that ϑ_3 corresponds to the third COM variable $p_{\beta\max}$ and the rest two are V and λ the particle velocity and the normalized magnetic moment.

To determine the relationships between the old variables and new ones we use the following equations

$$\sin\frac{\vartheta_3}{2} = \sigma\frac{p - p_{\max}}{p_{\max} - p_{\min}}, \tag{8}$$

$$p = g'\left(p_\beta^l\right)\left(p_\beta - p_\beta^l\right), \tag{9}$$

$$p_{\max} = g'\left(p_\beta^l\right)\left(p_{\beta\max} - p_\beta^l\right), \tag{10}$$

$$\cos(\xi + p_{\max})\sigma_m = \cos(\xi + p)\cos\psi, \tag{11}$$

where p_β and $p_{\beta\max}$ are the toroidal angular momentum in the frame moving along the torus with precession velocity in the axisymmetric toroidal field and it's maximum value along the superbanana orbit, $\sigma = \mathrm{sign}(\dot{p})$ and $\sigma_m = \cos(\psi(p_{\max}))$. The details of the transition to this frame one can find in [1].

It should be noted that Eq.(11) is obtained from Eq.(2) under the condition that Hamiltonian h conserves along the orbit. It should be mentioned that definition of cyclic variable ϑ_3 differs from that in [1] and is similar to the definition introduced in [6, 7]. Note also that ϑ_3 is not the canonical variable, however, it is more convenient for envisaged averaging procedure along the orbit.

In order to find the maximum value of variable p along the orbit one should calculate derivative $\partial_\psi p$ assuming $h = const$ along the orbit. As a result the equation for $p_{\max}$ is as follows

$$\left.\frac{\partial p}{\partial \psi}\right|_{h=const} = 0, \tag{12}$$

and can be rewritten in the explicate form as

$$\left.\frac{\partial p}{\partial \psi}\right|_{h=const} \equiv -\frac{\cos(\xi + p)\sin\psi}{\sin(\xi + p)\cos\psi} = 0. \tag{13}$$

Eq. (13) has two groups of roots

$$p_{m1} = -\xi + \frac{\pi}{2} + \pi m_1, \text{ where } \psi_{m1} \neq \frac{\pi}{2} + \pi m_1', \tag{14}$$

$$\psi_{m2} = \pi m_2, \text{where } p_{m2} \neq -\xi + \pi m_2', \tag{15}$$

with m_1, m_1', m_2 and $m_2' \in \mathbb{Z}$.

Analyzing the sign of $\left.\frac{\partial^2 p}{\partial \psi^2}\right|_{\substack{h=const \\ (p_m,\psi_m)}}$ it is possible to separate roots corresponding to minimum and to maximum values of p.

As far as

$$\left.\frac{\partial^2 p}{\partial \psi^2}\right|_{\substack{h=const \\ (p_m,\psi_m)}} = -\frac{\cos(\xi + p_m)}{\sin(\xi + p_m)}\frac{1}{\cos^2\psi_m}, \tag{16}$$

the roots (14) are associated with the points of inflection, and correspond to maximum of p if $\xi + p_{m2} \in (\pi m_2'; \pi/2 + \pi m_2')$ and correspond respectively to minimum of p if $\xi + p_{m2} \in (\pi/2 + \pi m_2'; \pi + \pi m_2')$.

SUPERBANANA FOKKER-PLANCK EQUATION

In variables $\mathbf{c}' = (V, \lambda, p_{\beta\max}, \vartheta_3)$, where V and λ are the magnitude of particle velocity and normalized magnetic moment [1], the Fokker-Planck equation can be represented as

$$\left(\partial_t + \dot{\vartheta}_3 \partial_{\vartheta_3}\right) f = Cf + S(\mathbf{c}'), \tag{17}$$

with collision term C in the form

$$C = \sqrt{g_{\mathbf{c}'}^{-1}} \left\{ \sum_{i,j\le 3} \partial_{c'^i} \left[\sqrt{g_{\mathbf{c}'}} \left(d_{\mathbf{c}'}^i - D_{\mathbf{c}'}^{ij} \partial_{c'^j} \right) \right] + \partial_{\vartheta_3} \left[\sqrt{g_{\mathbf{c}'}} \left(d_{\mathbf{c}'}^4 - D_{\mathbf{c}'}^{44} \partial_{\vartheta_3} - \sum_{i\le 3} D_{\mathbf{c}'}^{i4} \partial_{c'^i} \right) \right] \right\}, \tag{18}$$

where

$$d_{c'}^i = d_c^j, \quad D_{c'}^{ij} = D_c^{ij}, \quad D_{c'}^{il} = D_c^{ik}\frac{\partial c'^l}{\partial c^k} \quad \text{if} \quad i,j = 1,2; \quad k = 1,2,3,4; \quad l = 3,4 \quad \text{and} \quad d_{c'}^i = d_c^k \frac{\partial c'^i}{\partial c^k},$$
$$D_{c'}^{ij} = D_c^{kl}\frac{\partial c'^i}{\partial c^k}\frac{\partial c'^j}{\partial c^l} \text{ if } i,j = 3,4;\ k,l = 1,2,3,4; \tag{19}$$

and $\mathbf{c}=\left(V,\lambda,p_\beta,\psi\right)$ denotes the set of the reference axisymmetric coordinates.

Thus to obtain the explicit analytical expression for collision operator C one should calculate the derivatives $\frac{\partial c'^l}{\partial c^k}$ which in the limit of strong toroidal field ripple are given by the following expressions

$$\frac{\partial p_{\beta\max}}{\partial p_\beta}=\frac{\sin\left(\xi+p\right)\cos\psi}{\sigma_m\sin\left(\xi+p_{\max}\right)}, \tag{20}$$

$$\frac{\partial p_{\beta\max}}{\partial\psi}=\frac{1}{g'}\frac{\cos\left(\xi+p\right)\sin\psi}{\sigma_m\sin\left(\xi+p_{\max}\right)}, \tag{21}$$

$$\frac{\partial\vartheta_3}{\partial p_\beta}=\frac{2g'\sigma}{\Delta\cos\frac{\vartheta_3}{2}}\left\{1-\frac{\partial p_{\beta\max}}{\partial p_\beta}-\frac{p-p_{\max}}{\Delta}\left(1-\frac{\sigma_m\sin\left(\xi+p_{\max}\right)}{\sigma'_m\sin\left(\xi+p_{\min}\right)}\right)\frac{\partial p_{\beta\max}}{\partial p_\beta}\right\}, \tag{22}$$

$$\frac{\partial\vartheta_3}{\partial\psi}=\frac{2\sigma}{\cos\frac{\vartheta_3}{2}}\left\{-\frac{\partial p_{\beta\max}}{\partial\psi}-\frac{p-p_{\max}}{\Delta}\left(1-\frac{\sigma_m\sin\left(\xi+p_{\max}\right)}{\sigma'_m\sin\left(\xi+p_{\min}\right)}\right)\frac{\partial p_{\beta\max}}{\partial\psi}\right\}, \tag{23}$$

$$\frac{\partial p_{\beta\max}}{\partial\left(\upsilon,\lambda\right)}\approx\left(1-\frac{\partial p_{\beta\max}}{\partial p_\beta}\right)\frac{\partial p_\beta^l}{\partial\left(\upsilon,\lambda\right)}, \tag{24}$$

$$\frac{\partial\vartheta_3}{\partial\left(\upsilon,\lambda\right)}\approx-\frac{\partial\vartheta_3}{\partial p_\beta}\frac{\partial p_\beta^l}{\partial\left(\upsilon,\lambda\right)}. \tag{25}$$

Expressions (20)-(25) allow to construct the friction force $\mathbf{d}$ and the diffusion tensor $\overleftrightarrow{\mathbf{D}}$, describing correspondingly the convective and diffusive collisional transport of fast ions in a tokamak with the strong TF ripples.

SUMMARY AND DISCUSSION

The transport coefficients of Fokker-Planck equation describing collisional transport of fast ions in tokamaks with the strong TF ripples are derived.

Analysis of the superbanana orbits in the limit of the strong ripples is carried out. It is shown that analysis of orbits in the phase space could be restricted to analysis of orbits in two domains with the different signs of the corresponding adiabatic invariant.

Transition from the set of COM variables in axisymmetric case to the set of COM variables in ripple perturbed case is done for the strong ripple limit. Expressions for friction force $\mathbf{d}$ and diffusion tensor $\overleftrightarrow{\mathbf{D}}$, describing correspondingly the convective and diffusive collisional transport of fast ions in a tokamak with the strong TF ripples are obtained.

These equations can be used for the kinetic modelling of confined and lost energetic ions in modern and future tokamaks with rippled toroidal magnetic field.

REFERENCES

1. Yavorskij V.A., Andrushchenko Zh.N., Edenstrasser J.W, Goloborod'ko V.Ya. Three-dimensional Fokker-Planck equation for trapped fast ions in a Tokamak with weak toroidal field ripples // Physics of Plasmas. – 1999. – Vol.6, № 10. – P. 3853-3867.
2. Heidbrink W.W., Sadler G.J. The behaviour of fast ions in tokamak experiments // Nuclear Fusion. – 1994. – Vol.34, № 4. – P. 535-615.
3. Zweben S.J., Budny R.V., Darrow D.S., Medley S.S., Nazikian R. et.al. Alpha particle physics experiments in the Tokamak Fusion Test Reactor // Nuclear Fusion. – 2000. – Vol.40, № 1. – P. 91-148.
4. Bernstein I.B., Molvig K. Lagrangian formulation of neoclassical transport theory // Physics of Fluids. – 1983. – Vol.26, № 6. – P. 1488-1507.
5. Goloborod'ko V.Ya., Yavorskij V.A. Kinetic equation for high-energy particles of axisymmetric tokamaks in terms of constants of motion // Fusion Technology. – 1990. – Vol.18, № 11. – P. 414-424.
6. Goloborod'ko V.Ya, Reznik S.N., Yavorskij V.A., Zweben S.Z. Fokker-Planck 3-D modeling of axisymmetric collisional of fusion products in TFTR // Nuclear Fusion. – 1995. – Vol.35, № 12. – P. 1523-1535.
7. Goloborod'ko V.Ya., Yavorskij V.A. On formulation of 3D boundary-value problem for the fast particle distribution function in axisymmetric tokamak plasmas // Ukrainian Journal of Physics. – 1997. – Vol. 42, № 2. – P. 143-152.
8. Yavorskij V.A., Edenstrasser J.W, Goloborod'ko V.Ya. Reznik S.N., Zweben S.Z. Fokker-Planck modelling of delayed loss of

charged fusion products in TFTR // Nuclear Fusion. – 1998. – Vol.38, № 10. – P. 1565-1576.

9. Yavorskij V.A., Schoepf K., Edenstrasser J.W, Goloborod’ko V.Ya. 3D Fokker Planck Model for High and Moderate Energy Ions in Weakly Ripple Tokamaks // 18th IAEA Fusion Energy Conference, Sorrento (Italy), October 4-10. – 2000. – IAEA-THP2/30.
10. Fasoli A., Gormenzano C., Berk H.L., Breizman B., Briguglio S. et al Chapter 5: Physics of Energetic Ions // Nuclear Fusion. – 2007. – Vol.47, № 6. – P. S264-S284.
11. Goldston R.J., Towner H.H. Effects of toroidal field ripple on suprathermal ions in tokamak plasmas // Journal of Plasma Physics. – 1981. – Vol.26, № 02. – P. 283-307.
12. Goldston R.J., White R.B., Boozer A.H. Confinement of High-Energy Trapped Particles in Tokamaks // Physical Review Letters. – 1981. – Vol.47, № 9. – P. 647-649.
13. Yushmanov P.N. // Reviews of Plasma Physics (Edited by M. A. Leontovich, Published by Consultants Bureau New York, London). – 1987. – Vol 16. – P. 117-191.
14. Assas S., Eriksson L.-G., Nguyen F., Basiuk V. On the asymmetry of ripple induced losses in ICRF heated Tore Supra discharges // Plasma Physics and Controlled Fusion. – 2003. – Vol.45, № 2. – P. 145-158.
15. Salmi A., Johnson T., Parail V., Heikkinen J., Hynönen V. ASCOT Modelling of Ripple Effects on Toroidal Torque // Contributions to Plasma Physics. – 2008. – Vol.48, № 1-3. – P. 77-81.
16. Hideyuki M., Tsutsui H., Tsuji-iio S., Ryuichi S., Tani K. A Theoretical Model of Ripple Resonance Diffusion of Alpha Particles in Tokamaks(P1-35) // 18th International Toki Conference, Ceratopia Toki, Japan, December 9-12. – 2008. – ITC(18)-P1-35. – P. 261-264.
17. Hideyuki M., Tani K., Tsutsui H., Tobita K., Tsuji-iio S. et. al. Numerical Study of the Ripple Resonance Diffusion of Alpha Particles in Tokamaks // Plasma and Fusion Research. . – 2009. – Vol.4. – P. 008-1-008-8.
18. Saibene G., Sartori R., Mcdonald Dc, Beurskens M., Salmi A et. al. Results of the variable toroidal field ripple experiments in JET // 22th IAEA Fusion Energy Conference, Geneva, October 13-18. – 2008. – IAEA-CN-165/EX/2-1.
19. Vries P.C., Salmi A., Parail V., Giroud C., Andrew Y. et. al. Effect of toroidal field ripple on plasma rotation in JET // Nuclear Fusion. – 2008. – Vol.48, № 3. – P. 1-6.
20. Putvinski S.V. Physics of Energetic Particles in ITER // Nuclear Fusion. – 1998. – Vol.38, № 9. – pages 1275-1281.
21. Kurki-Suonio T., Asunta O., Hynönen V., Johnson T., Koskela T. Fast Particle Losses in ITER // 22th IAEA Fusion Energy Conference, Geneva, October 13-18. – 2008. – IAEA-CN-165/IT/P6-2.
22. Kramer G.J., White R.B., Nazikian R., Berk H.L. Fusion-born Alpha Particle Ripple Loss Studies in ITER // Princeton Plasma Physics Laboratory Preprint. – 2009. – March. – PPPL-4363.
23. Yavorskij V.A., Moskvitin A.A., Moskvitina Yu.K., Goloborod’ko V., Schoepf K. Fokker-Planck description of TF ripple induced collisional transport of fast ions in tokamaks // 11th IAEA Technical Meeting on “Energetic Particles in Magnetic Confinement Systems” Kyiv, Ukraine, September 21-23. – 2009. – IAEA-P23, Book of Abstracts, p.73.